

# Features of Marx's Solution

1. Total profit = total surplus value
2. Total value = total price
3.  $r_V = r_P$  for economy as a whole
4.  $s/v = \Pi/W$  for economy as a whole
5. The only differences between V and P systems is at the level of individual industries, where

$$\Pi_j \neq S_j$$

$$r_P \neq r_V$$

## Features of Bortkiewicz Solution

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1. Can require either
  - a) Total profit = total surplus value, or
  - b) Total value = total price
2.  $r_v \neq r_p$  in general for economy as a whole
3.  $r_p > 0$  if and only if  $r_v > 0$  (Fundamental Marxian Theorem)
4. With given conditions of production (unchanging  $c/v$  in each industry),  $r_v \uparrow$  iff  $r_p \uparrow$  and  $r_v \downarrow$  iff  $r_p \downarrow$ .

# New Solution to the Transformation Problem

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Key assumption of Bortkiewicz solution: real wage is given

New Solution substitutes two other assumptions:

- 1) the "money wage" is given and is the same in the V and P systems
- 2) the "monetary expression of labor time" is given and is the same in V and P systems

The New Solution is a different solution to the transformation problem since it defines the relation between the value system and the price system differently.

# Monetary Expression of Labor Time

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Define the "monetary expression of labor time" (MELT) as follows:

$$\mu = (\text{money value of net output}) / (\text{total new labor hours expended})$$

Example: If  $\mu = \$10/\text{hour}$ , that means one hour of social labor creates \$10 of value.

## Value of Labor-Power

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V System:  $V_{LP} = \sum \lambda_i b_i$  (in labor hours)

Money wage (same in V & P systems) must cover money cost of  $V_{LP}$ .

Thus,  $w = \mu \sum \lambda_i b_i$  (which can be thought of as  $V_{LP}$  in money terms in V system).

Thus,  $V_{LP} = w/\mu$ .

P System:  $w = \sum P_i b_i$

Later will show that the  $\{b_i\}$  above are different in the V system and P system.

## Setting up the Transformation

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First assumption: Money wage is given and is the same in the V and P systems:

Thus, find a set of prices of production that equalize profit rates in all industries, assuming that the money wage rate is the same in the V and P systems:

$$P_j = (1+r) [(\sum P_i a_{ij} + wL_j)] \quad (1)$$

This defines n equations which can be expressed in matrix notation as follows:

$$\mathbf{P} = (1+r)(\mathbf{PA} + w\mathbf{L}) \quad (1')$$

## Setting up the Transformation (cont)

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$$P_j = (1+r) [(\sum P_i a_{ij} + wL_j)] \quad (1)$$

$$\mathbf{P} = (1+r)(\mathbf{PA} + w\mathbf{L}) \quad (1')$$

Note that by replacing  $\sum P_i b_i$  by  $w$ , we are keeping  $w$  the same in the  $V$  and  $P$  systems.

## Setting up the Transformation (cont)

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Second assumption: The MELT is the same in the V and P systems:

To derive the (n+1)st equation, we equate the monetary expression of labor time in the V and P systems:

$$\mu_P = \mu_V.$$



## Setting up the Transformation (cont)

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By definition, we have

$$\mu_P = (\text{net value of output in price terms}) / \text{total new labor}$$

$\mathbf{PQ} - \mathbf{PAQ} = \mathbf{P(I - A)Q}$  is the matrix expression for net output in the price system.

$\mathbf{LQ}$  is the matrix expression for total new labor.

Thus,

$$\mu_P = \mathbf{P(I - A)Q/LQ} \quad (2)$$

## Setting up the Transformation (cont)

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By definition,

$\mu_V = P^V_j / \lambda_j$  for any value of  $j$  (this ratio is the same for all commodities in the  $V$  system).

$P^V_j$  is the value of a commodity in the  $V$  system expressed in money units.

The above equation tells us that we can regard  $\mu_V$  as given (we know it from the value system alone).

## Setting up the Transformation (cont)

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Since we assume  $\mu_P = \mu_V$ , the two  $\mu$ 's are equal and we can drop the subscripts and simply write  $\mu$  for either of them.

Hence, we can rewrite equation (2) as follows:

$$\mu_P = \mathbf{P}(\mathbf{I} - \mathbf{A})\mathbf{Q}/\mathbf{LQ} \quad (2)$$

$$\mathbf{P}(\mathbf{I} - \mathbf{A})\mathbf{Q} = \mu\mathbf{LQ}. \quad (3)$$

Equation (3) is the  $(n+1)$ st equation, which keeps the MELT the same in the V and P systems.

## Setting up the Transformation (cont)

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$$\mathbf{P} = (1+r)(\mathbf{PA} + w\mathbf{L}) \quad (1')$$

$$\mathbf{P}(\mathbf{I} - \mathbf{A})\mathbf{Q} = \mu\mathbf{LQ}. \quad (3)$$

Combining equations (1') and (3), we have  $n+1$  equations in the  $n+1$  unknowns  $P_1, \dots, P_n, r$ .

Under reasonable assumptions, these equations can be solved for non-negative values of those unknowns, with  $a_{ij}, L_j, Q_j, w$ , and  $\mu$  given.

# Features of the New Solution

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1. Total profit equals total surplus value (the latter measured in money terms).

$$\Pi = y - W = \mathbf{P(I-A)Q} - w\mathbf{LQ}$$

$$S_{\S} = \mu\mathbf{LQ} - w\mathbf{LQ}$$

The (n+1)st equation was  $\mathbf{P(I - A)Q} = \mu\mathbf{LQ}$

Hence,  $\Pi = S$

## Features of the New Solution (con't)

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1. Total profit equals total surplus value (the latter measured in money terms).
2. The total wage bill equals total variable capital (the latter measured in money terms.)
3. Therefore,  $\Pi/W = s/v$ .
4. The real wage is different in the V and P systems.
5.  $r_p$  is not equal to  $r_v$ , and they could still move in opposite directions.

## Real Wage Different in V and P Systems

Using notation  $\{b_i^V\}$  for worker's consumption basket in V system and  $\{b_i^P\}$  for worker's consumption basket in P system:

$$\text{In V system: } w = \mu \sum \lambda_i b_i^V = \sum (\mu \lambda_i) b_i^V$$

$$\text{In P System: } w = \sum P_i b_i^P \quad (2)$$

Thus,  $\{b_i^V\}$  is the same as  $\{b_i^P\}$  if and only if  $P_i = \mu \lambda_i$  for all  $i$  – that is, if price equals value for all commodities.

## Claims of Advocates of New Solution

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1. It shows that profits in the aggregate derive from surplus labor time (unpaid labor).
2. It justifies doing empirical work using Marxian value categories.

Note: Advocates of this solution do not present value as the basis of exchange ratios.



# The New Interpretation of Marxian Value Theory

The New Solution to the transformation problem evolved into the New Interpretation of Marxian value theory, as follows (since the monetary expression of labor time,  $\mu$ , is redefined below, we will call it  $\mu^*$ ):

$$\mu^* \equiv \text{net output in market prices} / \text{total new labor expended} = y^*/L$$

where  $y^*$  = net national product at market prices and  $L$  = total new labor expended in the economy.

Variable capital is defined as

$$v^* \equiv W^*/\mu^*, \text{ where } W^* \text{ is actual total money wages paid.}$$

## New Interpretation of Marxian Value Theory (cont)

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Thus, based on those definitions, total surplus value is reinterpreted as the labor hours measure of *actual total profit* (regardless of the way in which prices are determined):

If  $S_{\$}$  = surplus value in money units, we have

$$S_{\$} = \mu^*(L - v^*) = y^* - W^* = \Pi$$

No separate V and P systems in New Interpretation, just one system.

The value system based on embodied labor hours in commodities is dispensed with.

# Critics View of the New Solution/New Interpretation

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1. New Interpretation gives up key insights of MVT regarding surplus as appropriation of labor by capital. Instead, it is just an arbitrary division of actual output and profits by the number of labor hours.
2. New Interpretation reverses the relation between V and P.

The value categories are derived from price categories rather than vice versa.

Surplus value becomes a form of profit.

# Critics View of the New Solution/New Interpretation

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3. In New Solution, some traditional MVT principles no longer hold.

For example: If capitalists change their specific consumption choices, can show the following consequences:

P structure changes

MELT changes

value of labor power changes

s/v changes

## Critics View of the New Solution/New Interpretation

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4. Little is gained from the New Solution -- only  $s/v = \Pi/W$ .

In Bortkiewicz solution it is already the case that the prices of production reallocate a pre-existing amount of surplus value.

It is not obvious how the New Solution justifies empirical work using Marxian value categories in a way that the Bortkiewicz solution would not.

The New Interpretation "justifies" doing empirical work but by abandoning value categories in their usual meaning.